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## Research Article

# Cosmic Baryon Asymmetry in Different Neutrino Mass Models with Mixing Angles

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We investigate the comparative studies of cosmological baryon asymmetry in different neutrino mass models with and without  $\theta_{13}$  by considering the three-diagonal form of Dirac neutrino mass matrices and the three aspects of leptogenesis, unflavoured, flavoured, and nonthermal. We found that the estimations of any models with  $\theta_{13}$  are consistent in all the three stages of calculations of leptogenesis and the results are better than the predictions of any models without  $\theta_{13}$  which are consistent in a piecemeal manner with the observational data in all the three stages of leptogenesis calculations. For the normal hierarchy of Type-IA with charged lepton matrix, model with and without  $\theta_{13}$  predicts inflaton mass required to produce the observed baryon asymmetry to be  $M_\phi \sim 2.2 \times 10^{11}$  GeV and  $M_\phi \sim 3.6 \times 10^{10}$  GeV, and the corresponding reheating temperatures are  $T_R \sim 4.86 \times 10^6$  GeV and  $T_R \sim 4.50 \times 10^6$  GeV respectively. These predictions are not in conflict with the gravitino problem which required the reheating temperature to be below  $10^7$  GeV. And these values apply to the recent discovery of Higgs boson of mass  $\sim 125$  GeV. One can also have the right order of relic dark matter abundance only if the reheating temperature is bounded to below  $10^7$  GeV.

## 1. Introduction

Recent measurement of a moderately large value of the third mixing angle  $\theta_{13}$  by reactor neutrino oscillation experiments around the world particularly by Daya Bay ( $\sin^2\theta_{13} = 0.089 \pm 0.010(\text{stat}) \pm 0.005(\text{syst})$ ) [1] and RENO ( $\sin^2\theta_{13} = 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{syst})$ ) [2] signifies an important breakthrough in establishing the standard three-flavour oscillation picture of neutrinos. Thereby, we will address the issues of the recent indication of nonmaximal 2-3 mixing by MINOS accelerator experiment [3] leading to determining the correct octant of  $\theta_{23}$  and neutrino mass hierarchy. Furthermore, now, this has opened the door to study leptonic CP violation in a convincing manner, which in turn has profound implications for our understanding of the matter-antimatter asymmetry of the universe. In fact, ascertaining the origin of the cosmological baryon asymmetry,  $\eta_B = (6.5^{+0.4}_{-0.5}) \times 10^{-10}$  [4], is one of the burning open issues in both particle physics and cosmology. The asymmetry must have been generated

during the evolution of the universe. However, it is possible to dynamically generate such asymmetry if three conditions, (i) the existence of baryon number violating interactions, (ii) C and CP violations, and (iii) the deviation from thermal equilibrium, are satisfied [5]. There are different mechanisms of baryogenesis, but leptogenesis [6] is attractive because of its simplicity and the connection to neutrino physics. Establishing a connection between the low-energy neutrino mixing parameters and high-energy leptogenesis parameters has received much attention in recent years in [6–9]. In leptogenesis, the first condition is satisfied by the Majorana nature of heavy neutrinos and the sphaleron effect in the standard model (SM) at the high temperature [9], while the second condition is provided by their CP-violating decay. The deviation from thermal equilibrium is provided by the expansion of the universe. Needless to say the departures from thermal equilibrium have been very important without it; the past history of the universe would be irrelevant, as the present state would be merely that of a system at 2.75 K, very

uninteresting indeed [10]. One of the keys to understanding the thermal history of the universe is the estimation of cosmological baryon asymmetry from different neutrino mass models with the inclusion of the latest nonzero  $\theta_{13}$ .

Broadly the leptogenesis can be grouped into two groups: thermal with and without flavour effects and nonthermal leptogenesis. The simplest scenario, namely, the standard thermal leptogenesis, requires nothing but the thermal excitation of heavy Majorana neutrinos which generate tiny neutrino masses via the seesaw mechanism [11–13] and provides several implications for the light neutrino mass spectrum [14, 15]. And with heavy hierarchical right-handed neutrino spectrum, the CP asymmetry and the mass of the lightest right-handed Majorana neutrino are correlated. In order to have the correct order of light neutrino mass-squared differences, there is a lower bound on the mass of the right-handed neutrino,  $M_N \geq 10^9$  GeV [16–19], which in turn put constraints on reheating temperature after inflation to be  $T_R \geq 10^9$  GeV. This will lead to an excessive gravitino production and conflicts with the observed data. In the postinflation era, these gravitinos are produced in a thermal bath due to annihilation or scattering processes of different standard particles. The relic abundance of gravitino is proportional to the reheating temperature of the thermal bath. One can have the right order of relic dark matter abundance only if the reheating temperature is bounded to below  $10^7$  GeV [8, 20–24]. On the other hand, big-bang nucleosynthesis in SUSY theories also sets a severe constraint on the gravitino mass and the reheating temperature leading to the upper bound  $T_R \geq 10^7$  GeV [25–29]. While thermal leptogenesis in SUSY SO(10) with high seesaw scale easily satisfies the lower bound, the tension with the gravitino constraint is manifest.

According to Fukuyama et al. [30, 31], the nonthermal leptogenesis scenario in the framework of a minimal supersymmetric SO(10) model with Type-I seesaw shows that the predicted inflaton mass needed to produce the observed baryon asymmetry of the universe is found to be  $M_\phi \sim 5 \times 10^{11}$  GeV for the reheating temperature  $T_R = 10^6$  GeV and weak scale gravitino mass  $m_{3/2} \sim 100$  GeV without causing the gravitino problem. It also claims that even if these values are relaxed by one order of magnitude ( $m_{3/2} \leq 10$  TeV,  $T_R = 10^7$  GeV), the result is still valid. In [32, 33] using the Closed-Time-Path approach, they performed a systematic leading order calculation of the relaxation rate of flavour correlations of left-handed standard model leptons; and for flavoured leptogenesis in the early universe they found the reheating temperature to be  $T_R = 10^7$  GeV to  $10^{13}$  GeV. These values apply to the standard model with a Higgs-boson mass of 125 GeV [34]. The recent discovery of a standard model (SM) like Higgs boson provides further support for leptogenesis mechanism, where the asymmetry is generated by out-of-equilibrium decays of our conjecture heavy sterile right-handed neutrinos into a Higgs boson and a lepton. In [35] split neutrinos were introduced where there is one Dirac neutrino and two Majorana neutrinos with a slight departure from tribimaximal mixing (TBM), which explains the reactor angle  $\sim \theta_{13}$ , and tied intimately to the lepton asymmetry

and can explain inflation, dark matter, neutrino masses, and the baryon asymmetry, which can be further constrained by the searches of SUSY particles at the LHC, the right-handed sneutrino, essentially the inflaton component as a dark matter candidate, and from the  $0\nu\beta\beta$  experiments. In [36] too a deviation from TBM case was studied with model-independent discussion and the existing link between low- and high-energy parameters that connect to the parameters governing leptogenesis was analysed. However, in [37] exact TBM,  $\tan^2\theta_{12} = 0.50$ , was considered with charged lepton and up-quark type and set  $\theta_{13}$  to be zero; eventually their results differ from ours. We slightly modify the neutrino models in [37]; consequently the inputs parameters are different for zero  $\theta_{13}$  but for nonzero  $\theta_{13}$  our formalism is entirely different than the one done in [37]; besides we consider  $\tan^2\theta_{12} = 0.45$  for detail analysis. Our work in this paper is consistent with the values given in [30–35].

Now, the theoretical framework supporting leptogenesis from low-energy phases has some other realistic testable predictions in view of nonzero  $\theta_{13}$ . So the present paper is a modest attempt to compare the predictions of leptogenesis from low-energy CP-violating phases in different neutrino mass matrices with and without  $\theta_{13}$ . The current investigation is twofold. The first part deals with zero reactor mixing angle in different neutrino mass models within  $\mu$ - $\tau$  symmetry [38–49], while in the second part we construct  $m_{LL}$  matrix from fitting of  $U_{PMNS}$  incorporating the nonzero third reactor angle ( $\theta_{13}$ ) along with the observed data and subsequently predict the baryon asymmetry of the universe (BAU). We must also mention that there are several works analysing the link between leptogenesis and low-energy data in more general scenarios. However, we have not come across in the literature where all the three categories of leptogenesis, that is, the thermal leptogenesis with or without flavour effects and nonthermal leptogenesis, are studied in a single paper. Take, for instance, some of the major players working on leptogenesis. Professor Wilfried Buchmuller works are mostly confined to standard unflavoured thermal leptogenesis by solving Boltzmann's equation whereas Professor Steven Blanchet and Professor P. Di. Bari generally worked on flavoured effects in leptogenesis and lesser people work on nonthermal leptogenesis (cf. [30, 31]). But we attempt to study all the three aspects of leptogenesis in this paper, which makes our work apparently different from others on this account.

The detailed plan of the paper is as follows. In Section 2, methodology and classification of neutrino mass models for zero  $\theta_{13}$  are presented. Section 3 gives an overview of leptogenesis. The numerical and analytic results for neutrino mass models  $m_{LL}$  without and with  $\theta_{13}$  are given in Sections 4 and 5, respectively. We end with conclusions in Section 6.

## 2. Methodology and Classification of Neutrino Mass Models

We begin with Type-I seesaw mechanism for estimation of BAU. The required left-handed light neutrino mass models  $m_{LL}$  without  $\theta_{13}$  are given in Table 4. And  $m_{LL}$  can be related

to the right-handed Majorana mass matrix  $M_{RR}$  and the Dirac mass matrix  $m_{LR}$  through the inversion seesaw mechanism:

$$M_{RR} = -m_{LR}^T m_{LL}^{-1} m_{LR}, \quad (1)$$

where

$$m_{LR} = \text{diag}(\lambda^m, \lambda^n, 1) \nu. \quad (2)$$

In (2)  $(m, n)$  are two integers depending on the type of Dirac mass matrix we choose. Since the texture of Yukawa matrix for Dirac neutrino is not known, we take the diagonal texture of  $m_{LR}$  to be of charged lepton mass matrix (6, 2), up-quark type mass matrix (8, 4), or down-quark type mass matrix (4, 2), as allowed by SO(10) GUT models.

For computations of leptogenesis, we choose a basis  $U_R$  where  $M_{RR}^{\text{diag}} = U_R^T M_{RR} U_R = \text{diag}(M_1, M_2, M_3)$  with real and positive eigenvalues. And the Dirac mass matrix  $m_{LR}$  in the prime basis transforms to  $m'_{LR} \rightarrow m_{LR} U_R Q$ , where  $Q$  is the complex matrix containing CP-violating Majorana phases  $\phi_1$  and  $\phi_2$  derived from  $M_{RR}$ . The values of  $\phi_1$  and  $\phi_2$  are chosen arbitrarily other than  $\pi/2$  and 0. We then set the Wolfenstein parameter as  $\lambda = 0.3$  and compute the three choices of  $(m, n)$  in  $m_{LR}$ . In this prime basis the Dirac neutrino Yukawa coupling becomes  $h = m'_{LR}/\nu$  and subsequently this value is used in the expression of CP asymmetry. The new Yukawa coupling matrix  $h$  also becomes complex, and hence the term  $\text{Im}(h^\dagger h)_{ij}$  appearing in CP asymmetry parameter  $\epsilon_1$  gives a nonzero contribution.

In the second part of this paper, we construct  $m_{LL}$  from  $U_{\text{PMNS}}$  matrix with  $\theta_{13}$  value:

$$m_{LL} = U_{\text{PMNS}} \cdot m_{\text{diag}} \cdot U_{\text{PMNS}}^T, \quad (3)$$

where  $U_{\text{PMNS}}$  is the Pontecorvo-Maki-Nakagawa-Sakata parameterised matrix taken from the standard particle data group (PDG) [50], and the corresponding mixing angles are

$$\sin^2 \theta_{13} = |U_{e3}|^2, \quad \tan^2 \theta_{12} = \frac{|U_{e2}|^2}{|U_{e1}|^2}, \quad (4)$$

$$\tan^2 \theta_{23} = \frac{|U_{\tau 3}|^2}{|U_{\mu 3}|^2},$$

$$m_{\text{diag}} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & \pm m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \quad (5)$$

A global analysis [51, 52] current best-fit data is used in the present analysis:

$$\begin{aligned} \Delta m_{21}^2 &= 7.6 \times 10^{-5} \text{ eV}^2, & \Delta m_{31}^2 &= 2.4 \times 10^{-3} \text{ eV}^2, \\ \sin^2 \theta_{12} &= 0.312, & \sin^2 \theta_{23} &= 0.42, & \sin^2 \theta_{13} &= 0.025, \\ \theta_{12} &= 34^\circ \pm 1^\circ, & \theta_{23} &= 40.4^{+4.6}_{-1.8}, & \theta_{13} &= 9.0^\circ \pm 1.3^\circ. \end{aligned} \quad (6)$$

Neutrino oscillation data are insensitive to the low-energy individual neutrino masses. However, it can be measured in

tritium beta decay [53] and neutrinoless double beta decay [54] and from the contribution of neutrinos to the energy density of the universe [55]. Very recent data from the Planck experiment have set an upper bound over the sum of all the neutrino mass eigenvalues of  $\sum_{i=1}^3 m_i \leq 0.23 \text{ eV}$  at 95% C.L. [56]. But, oscillations experiments are capable of measuring the two independent mass-squared differences  $\Delta m_{21}^2 = m_2^2 - m_1^2$  and  $\Delta m_{31}^2 = m_3^2 - m_1^2$  only. This two flavours oscillation approach has been quite successful in measuring the solar and atmospheric neutrino parameters. In the future the neutrino experiments must involve probing the full three flavor effects, including the subleading ones proportional to  $\alpha = \Delta m_{21}^2 / |\Delta m_{31}^2|$ . The  $\Delta m_{21}^2$  is positive as is required to be positive by the observed energy dependence of the electron neutrino survival probability in solar neutrinos but  $\Delta m_{31}^2$  is allowed to be either positive or negative by the present data. Hence, two patterns of neutrino masses are possible:  $m_1 < m_2 \ll m_3$  called normal hierarchy (NH) where  $\Delta m_{31}^2$  is positive and  $m_3 \ll m_1 < m_2$  called inverted hierarchy (IH) where  $\Delta m_{31}^2$  is negative. A third possibility, where the three masses are nearly quasi-degenerate with very tiny differences,  $m_1 \leq m_2 \sim m_3$ , between them, also exists with two subcases of  $\Delta m_{31}^2$  being positive or negative.

Leptonic CP violation (LCPV) can be established if CP-violating phase  $\delta_{\text{CP}}$  is shown to differ from 0 to  $180^\circ$ . A detailed review on LCPV can be found in [57]. It was not possible to observe a signal for CP violation in the present data so far. Thus,  $\delta_{\text{CP}}$  can have any value in the range  $[-180^\circ, 180^\circ]$ . The Majorana phases  $\phi_1$  and  $\phi_2$  are free parameters. In the absence of constraints on the phases  $\phi_1$  and  $\phi_2$ , these have been given full variation between 0 and  $2\pi$  excluding these two extreme values.

### 3. Leptogenesis

As pointed out above leptogenesis can be thermal or nonthermal; again thermal leptogenesis can be unflavoured (single flavoured) or flavoured which are all explained in the subsequent pages. In the simplest form of leptogenesis the heavy Majorana neutrinos are produced by thermal processes, which is therefore called the ‘‘thermal leptogenesis.’’ For our estimations of CP asymmetry parameter  $\epsilon_1$  [6, 58, 59], we list here only the required equations for computations. However, interested reader can find more details in [60]. The low-energy neutrino physics is related to the high-energy leptogenesis physics through the seesaw mechanism. In (1),  $m_{LR}^T$  is the transpose of  $m_{LR}$  and  $m_{LL}^{-1}$  is the inverse of  $m_{LL}$ . For the third generation Yukawa coupling unification, in SO (10) grand unified theory, one obtains the heavy and light neutrino masses as  $M_3 \sim \Lambda_{\text{GUT}} \sim 10^{15} \text{ GeV}$  and  $m_3 \sim \nu^2/M_3 \sim 0.01 \text{ eV}$  respectively. Remarkably, the light neutrino mass  $m_3$  is compatible with  $(\Delta m_{\text{atm}}^2)^{1/2} \equiv m_{\text{atm}} \approx 0.05 \text{ eV}$ , as measured in atmospheric neutrino oscillations. This suggests that neutrino physics probes the mass scale of grand unification (GUT), although other interpretations of neutrino masses are possible as well. The heavy Majorana neutrinos have no gauge interactions. Hence, in the early universe, they can easily be out of thermal equilibrium.

This makes the lightest ( $N_1$ ) of the heavy right-handed Majorana neutrino an ideal candidate for baryogenesis, satisfying the third condition of Sarkar, the deviation from thermal equilibrium. Assuming hierarchical heavy neutrino masses ( $M_1 \ll M_2, M_3$ ), the CP asymmetry generated due to CP-violating out-of-equilibrium decay of  $N_1$  is given by

$$\epsilon_1 = \frac{\Gamma(N_R \rightarrow l_L + \phi) - \Gamma(N_R \rightarrow \bar{l}_L + \phi^\dagger)}{\Gamma(N_R \rightarrow l_L + \phi) + \Gamma(N_R \rightarrow \bar{l}_L + \phi^\dagger)}, \quad (7)$$

where  $\bar{l}_L$  is the antilepton of lepton  $l_L$  and  $\phi$  is the Higgs doublets chiral supermultiplets. Consider

$$\epsilon_1 = \frac{3}{16\pi} \left[ \frac{\text{Im}[(h^\dagger h)_{12}]}{(h^\dagger h)_{11}} \frac{M_1}{M_2} + \frac{\text{Im}[(h^\dagger h)_{13}]}{(h^\dagger h)_{11}} \frac{M_1}{M_3} \right], \quad (8)$$

where  $h = m_{LR}/\nu$  is the Yukawa coupling of the Dirac neutrino mass matrix in the diagonal basis of  $M_{RR}$  and  $\nu = 174 \text{ GeV}$  is the vev of the standard model. At high temperatures, between the critical temperature  $T_{EW}$  of the electroweak phase transition and a maximal temperature  $T_{SPH}$ ,

$$T_{EW} \sim 100 \text{ GeV} < T < T_{SPH} \sim 10^{12} \text{ GeV}, \quad (9)$$

these processes are believed to be in thermal equilibrium [9]. Although this important phenomenon is accepted by theorists as a correct explanation of baryogenesis via leptogenesis, it is yet to be tested experimentally. Therefore it is very fascinating that the corresponding phenomenon of chirality changing processes in strong interactions might be observed in heavy decay ion collisions at the LHC [61, 62]. The evolution of lepton number ( $L$ ) and baryon number ( $B$ ) is given by a set of coupled equations [63] by the electroweak sphaleron processes which violates  $(B+L)$  but conserves  $(B-L)$ . At temperature  $T$  above the electroweak phase transition temperature  $T_C$ , the baryon asymmetry can be expressed in terms of  $(B-L)$  number density as [64]

$$B(T > T_C) = \frac{8N_F + 4N_H}{22N_F + 13N_H} (B-L), \quad (10)$$

where  $(B-L)$  asymmetry per unit entropy is just the negative of the ratio of lepton density  $n_L$  and entropy ( $s$ ), since the baryon number is conserved in the right-handed Majorana neutrino decays. At  $T_C$ , any primordial  $(B+L)$  will be washed out and (10) can be written as [64, 65]

$$\frac{n_B}{s} \simeq -\frac{8N_F + 4N_H}{22N_F + 13N_H} \frac{n_L}{s}. \quad (11)$$

For standard model (SM) the number of fermion families  $N_F = 3$ , and the number of Higgs doublets  $N_H = 1$ ; and (11) reduces to

$$\frac{n_B}{s} \simeq -\frac{28}{79} \frac{n_L}{s}. \quad (12)$$

The ratio of baryon to photon is not conserved due to variation of photon density per comoving volume [66] at

different epoch of the expanding universe. However, for very slow baryon number  $B$  nonconserving interactions, the ratio of baryon to entropy in a comoving volume is conserved. Considering the cosmic ray microwave background temperature  $T \simeq 2.3 \text{ K}$ , we have  $s = 7.04n_\gamma$ . Here  $n_\gamma$  is a photon number density. And finally the observed baryon asymmetry of the universe [67, 68] for the case of standard model is calculated from

$$\eta_B^{SM} = \left( \frac{\eta_B}{\eta_\gamma} \right)^{SM} \approx 0.98 \times 10^{-2} \times \kappa_1 \epsilon_1. \quad (13)$$

The efficiency or dilution factor  $\kappa_1$  describes the washout of the lepton asymmetry due to various lepton number violating processes, which mainly depends on the effective neutrino mass

$$\tilde{m}_1 = \frac{(h^\dagger h)_{11} \nu^2}{M_1}, \quad (14)$$

where  $\nu$  is the electroweak vev;  $\nu = 174 \text{ GeV}$ . For  $10^{-2} \text{ eV} < \tilde{m}_1 < 10^{-3} \text{ eV}$ , the washout factor  $\kappa_1$  can be well approximated by [69]

$$\kappa_1(\tilde{m}_1) = 0.3 \left[ \frac{10^{-3}}{\tilde{m}_1} \right] \left[ \log \frac{\tilde{m}_1}{10^{-3}} \right]^{-0.6}. \quad (15)$$

We adopt a single expression for  $\kappa_1$  valid only for the given range of  $\tilde{m}_1$  [69–73]. And the comparison of the effective neutrino mass  $\tilde{m}_1$  with the equilibrium neutrino mass

$$m_* = \frac{8\pi H \nu^2}{M_1^2} \sim 1.1 \times 10^{-3} \text{ eV} \quad (16)$$

gives the information whether the system is weak or strong washout regime. For the weak washout regime we have  $\tilde{m}_1 < m_*$  and  $M_1 \geq 10^{12} \text{ GeV}$  whereas for the strong washout regime we have  $\tilde{m}_1 > m_*$  and  $M_1 \leq 10^{12} \text{ GeV}$ . However, the strong washout regime appears to be favoured by the present evidence for neutrino masses.

In the flavoured thermal leptogenesis [74–77], we look for enhancement in baryon asymmetry over the single flavour approximation and the equation for CP asymmetry in  $N_1 \rightarrow l_\alpha \phi$  decay where  $\alpha = (e, \mu, \tau)$  becomes

$$\begin{aligned} \epsilon_{\alpha\alpha} = & \frac{1}{8\pi} \frac{1}{(h^\dagger h)_{11}} \\ & \times \left( \sum_{j=2,3} \text{Im} \left[ h_{\alpha 1}^* (h^\dagger h)_{1j} h_{\alpha j} \right] g(x_j) \right. \\ & \left. + \sum_j \text{Im} \left[ h_{\alpha 1}^* (h^\dagger h)_{j1} h_{\alpha j} \right] \frac{1}{(1-x_j)} \right), \end{aligned} \quad (17)$$

where  $x_j = M_j^2/M_i^2$  and  $g(x_j) \sim (3/2)(1/\sqrt{x_j})$ . The efficiency factor is given by  $\kappa = m_*/\tilde{m}_{\alpha\alpha}$ . Here too  $m_* = (8\pi H \nu^2/M_1^2) \sim 1.1 \times 10^{-3} \text{ eV}$  and  $\tilde{m}_{\alpha\alpha} = (h_{\alpha 1}^* h_{\alpha 1}/M_1) \nu^2$ . This leads to the BAU:

$$\eta_{3B} = \frac{\eta_B}{\eta_\gamma} \sim 10^{-2} \sum_\alpha \epsilon_{\alpha\alpha} \kappa_\alpha \sim 10^{-2} m_* \sum_\alpha \frac{\epsilon_{\alpha\alpha}}{\tilde{m}_{\alpha\alpha}}. \quad (18)$$



TABLE 1: Predicted values of the solar and atmospheric neutrino mass-squared differences and mixing angles for  $\tan^2\theta_{12} = 0.45$ .

Type	$\Delta m_{21}^2$ ( $10^{-5} \text{ eV}^2$ )	$\Delta m_{21}^2$ ( $10^{-3} \text{ eV}^2$ )	$\tan^2\theta_{12}$	$\tan^2\theta_{23}$	$\sin\theta_{13}$
(IA)	7.82	2.20	0.45	1.0	0.0
(IB)	7.62	2.49	0.45	1.0	0.0
(IC)	7.62	2.49	0.45	1.0	0.0
(IIA)	7.91	2.35	0.45	1.0	0.0
(IIB)	8.40	2.03	0.45	1.0	0.0
(IIC)	7.53	2.45	0.45	1.0	0.0
(III)	7.61	2.42	0.45	1.0	0.0

For single flavour case, the second term in  $\epsilon_{\alpha\alpha}$  vanishes when summed over all flavours. Thus

$$\epsilon_1 \equiv \sum_{\alpha} \epsilon_{\alpha\alpha} = \frac{1}{8\pi} \frac{1}{(h^\dagger h)_{11}} \sum_j \text{Im} \left[ (h^\dagger h)_{1j}^2 \right] g(x_j); \quad (19)$$

this leads to baryon symmetry:

$$\eta_{1B} \approx 10^{-2} m_* \frac{\epsilon_1}{\tilde{m}} = 10^{-2} \kappa_1 \epsilon_1, \quad (20)$$

where  $\epsilon_1 = \sum_{\alpha} \epsilon_{\alpha\alpha}$  and  $\tilde{m} = \sum_{\alpha} \tilde{m}_{\alpha\alpha}$ . The conditions of weak or strong washout regime for flavoured leptogenesis are the same as in the case of single flavoured/unflavoured leptogenesis, however, with one difference that  $\tilde{m}_1$  is the effective mass due to unflavoured leptogenesis while  $\tilde{m}$  is the resultant effective mass due to contributions of three leptons (flavoured leptogenesis).

In nonthermal leptogenesis [78–83] the right-handed neutrinos  $N_i$  ( $i = 1, 2, 3$ ) with masses ( $M_1, M_2, M_3$ ) produced through the direct nonthermal decay of the inflaton  $\phi$  interact only with leptons and Higgs through Yukawa couplings. The inflaton decay rate  $\Gamma_{\phi}$  is given by [30]

$$\Gamma_{\phi} = \Gamma(\phi \rightarrow N_i N_i) \approx \frac{|\lambda|^2}{4\pi} M_{\phi}, \quad (21)$$

where  $M_{\phi}$  is the mass of inflaton  $\phi$ . The reheating temperature ( $T_R$ ) after inflation is [84]

$$T_R = \left( \frac{44}{2\pi^2 g} \right)^{1/4} (\Gamma_{\phi} M_P)^{1/2} \quad (22)$$

and the produced baryon asymmetry of the universe can be calculated by the following relation [85]:

$$Y_B = \frac{n_B}{s} = CY_L = C \frac{3}{2} \frac{T_R}{M_{\phi}} \epsilon, \quad (23)$$

where  $s = 7.0n_{\gamma}$  is related to  $Y_B = n_B/s = 8.7 \times 10^{-11}$  in (23). From (23) the connection between  $T_R$  and  $M_{\phi}$  is expressed as

$$T_R = \left( \frac{2Y_B}{3C\epsilon} \right) M_{\phi}. \quad (24)$$

Two boundary conditions in nonthermal leptogenesis are  $M_{\phi} > 2M_1$  and  $T_R \leq 0.01M_1$ . The values of  $M_1$  and  $\epsilon$  for all neutrino mass models are also used in the calculation of theoretical bounds:  $T_R^{\min} < T_R \leq T_R^{\max}$  and  $M_{\phi}^{\min} < M_{\phi} < M_{\phi}^{\max}$ . Only those models which satisfy these constraints can survive in the nonthermal leptogenesis.

#### 4. Numerical Analysis and Results without $\theta_{13}$

We first begin our numerical analysis for  $m_{LL}$  without  $\theta_{13}$  given in the Appendix. The predicted parameters for  $\tan^2\theta_{12} = 0.45$ , given in Table 1, are consistent with the global best-fit value. For computations of leptogenesis, we employ the well-known inversion seesaw mechanism as explained in Section 2. Finally the estimated BAU for both unflavoured  $\eta_{1B}$  and flavoured  $\eta_{3B}$  leptogenesis for  $m_{LL}$  without  $\theta_{13}$  is tabulated in Table 2. As expected, we found that there is an enhancement in BAU in the case of flavoured leptogenesis  $\eta_{3B}$  compared to unflavoured  $\eta_{1B}$ . We also observe the sensitivity of BAU predictions on the choice of models without  $\theta_{13}$  and all but the five models are favourable with good predictions (see Table 2). Streaming lining further, by taking the various constraints into consideration, quasi-degenerate Type-1A, QD-1A (6, 2), and NH-III (8, 4) are competing with each other, which can be tested for discrimination in the next level, the nonthermal leptogenesis.

In case of nonthermal leptogenesis, the lightest right-handed Majorana neutrino mass  $M_1$  and the CP asymmetry parameter  $\epsilon_1$  are taken from Table 2 and used in all the neutrino mass models  $m_{LL}$  while computing the bounds  $T_R^{\min} < T_R \leq T_R^{\max}$  and  $M_{\phi}^{\min} < M_{\phi} \leq M_{\phi}^{\max}$  and the computed results are tabulated in Table 3. The baryon asymmetry  $Y_B = \eta_B/s$  is taken as input value from WMAP observational data. If we compare these calculations with the predictions of certain inflationary models such as chaotic or natural inflationary model which predicts the inflaton mass to be  $M_{\phi} \sim 10^{13} \text{ GeV}$ , then from Table 3 the neutrino mass models with  $(m, n)$  which are compatible with  $M_{\phi} \sim 10^{13} \text{ GeV}$  are listed as IA-(4, 2), IIB-(4, 2), III-(4, 2), and III-(6, 2) only. The neutrino mass models with  $(m, n)$  should be compatible with  $M_{\phi} \sim (10^{10}-10^{13}) \text{ GeV}$ . Again in order to avoid gravitino problem [84] in supersymmetric models, one has the bound on reheating temperature,  $T_R \approx (10^6-10^7) \text{ GeV}$ . This constraint further streamlines the neutrino mass models and the accepted models are IA-(4, 2), IIB-(4, 2), and III-(6, 2) only.

Furthermore, on examination of the predictions of thermal leptogenesis (Table 2) and nonthermal leptogenesis (Table 3) we found that the estimated results are inconsistent with the two mechanisms of leptogenesis in spite of the fact that they are in agreement with the observation separately. Otherwise for a good model we expect these predictions to

TABLE 2: For zero  $\theta_{13}$ , the lightest RH Majorana neutrino mass  $M_1$  and values of CP asymmetry and baryon asymmetry for QDN models (IA, IB, and IC), IH models (IIA, IIB), and NH models (III), with  $\tan^2\theta_{12} = 0.45$ , using neutrino mass matrices given in Table 4. The entry  $(m, n)$  in  $m_{LR}$  indicates the type of Dirac neutrino mass matrix taken as charged lepton mass matrix (6, 2) or up-quark mass matrix (8, 4), or down-quark mass matrix (4, 2) as explained in the text. IA (6, 2) and III (8, 4) appear to be the best models.

Type	$(m, n)$	$M_1$	$\epsilon_1$	$\eta_{1B}$	$\eta_{3B}$	Status
(IA)	(4, 2)	$5.43 \times 10^{10}$	$1.49 \times 10^{-5}$	$7.03 \times 10^{-9}$	$2.16 \times 10^{-8}$	✓
(IA)	(6, 2)	$4.51 \times 10^8$	$1.31 \times 10^{-7}$	$5.76 \times 10^{-11}$	$1.34 \times 10^{-10}$	✓
(IA)	(8, 4)	$3.65 \times 10^6$	$1.16 \times 10^{-9}$	$5.72 \times 10^{-13}$	$1.19 \times 10^{-12}$	✗
(IB)	(4, 2)	$5.01 \times 10^9$	$2.56 \times 10^{-14}$	$7.15 \times 10^{-15}$	$1.09 \times 10^{-9}$	✗
(IB)	(6, 2)	$4.05 \times 10^7$	$2.06 \times 10^{-16}$	$5.76 \times 10^{-20}$	$8.84 \times 10^{-12}$	✗
(IB)	(8, 4)	$3.28 \times 10^5$	$1.68 \times 10^{-18}$	$4.67 \times 10^{-22}$	$7.16 \times 10^{-14}$	✗
(IC)	(4, 2)	$5.01 \times 10^9$	$1.85 \times 10^{-13}$	$5.12 \times 10^{-17}$	$7.16 \times 10^{-9}$	✗
(IC)	(6, 2)	$4.05 \times 10^7$	$1.47 \times 10^{-15}$	$3.77 \times 10^{-29}$	$5.80 \times 10^{-11}$	✗
(IC)	(8, 4)	$3.28 \times 10^5$	$1.02 \times 10^{-16}$	$2.82 \times 10^{-20}$	$4.34 \times 10^{-12}$	✗
(IIA)	(4, 2)	$4.02 \times 10^{10}$	$1.12 \times 10^{-12}$	$2.49 \times 10^{-15}$	$7.90 \times 10^{-11}$	✗
(IIA)	(6, 2)	$3.25 \times 10^8$	$9.00 \times 10^{-15}$	$2.00 \times 10^{-17}$	$6.34 \times 10^{-13}$	✗
(IIA)	(8, 4)	$2.63 \times 10^6$	$7.53 \times 10^{-17}$	$1.67 \times 10^{-19}$	$5.35 \times 10^{-15}$	✗
(IIB)	(4, 2)	$9.76 \times 10^{10}$	$4.02 \times 10^{-6}$	$3.25 \times 10^{-9}$	$7.53 \times 10^{-9}$	✗
(IIB)	(6, 2)	$8.10 \times 10^8$	$3.33 \times 10^{-8}$	$2.57 \times 10^{-11}$	$5.96 \times 10^{-11}$	✗
(IIB)	(8, 4)	$6.56 \times 10^6$	$2.71 \times 10^{-10}$	$2.09 \times 10^{-13}$	$4.86 \times 10^{-13}$	✗
(III)	(4, 2)	$3.73 \times 10^{12}$	$3.09 \times 10^{-5}$	$8.13 \times 10^{-8}$	$1.85 \times 10^{-6}$	✗
(III)	(6, 2)	$4.08 \times 10^{11}$	$3.74 \times 10^{-5}$	$7.37 \times 10^{-10}$	$1.62 \times 10^{-9}$	✓
(III)	(8, 4)	$3.31 \times 10^9$	$3.09 \times 10^{-7}$	$6.06 \times 10^{-11}$	$1.13 \times 10^{-10}$	✓

TABLE 3: Theoretical bound on reheating temperature  $T_R$  and inflaton masses  $M_\phi$  in nonthermal leptogenesis, for all neutrino mass models with  $\tan^2\theta_{12} = 0.45$ . Models which are consistent with observations are marked in the status column.

Type	$(m, n)$	$T_R^{\min} < T_R \leq T_R^{\max}$	$M_\phi^{\min} < M_\phi \leq M_\phi^{\max}$	Status
(IA)	(4, 2)	$1.2 \times 10^6 < T_R \leq 5.4 \times 10^8$	$1.1 \times 10^{11} < M_\phi \leq 4.9 \times 10^{13}$	✓
(IA)	(6, 2)	$1.1 \times 10^6 < T_R \leq 4.5 \times 10^6$	$9.0 \times 10^8 < M_\phi \leq 3.6 \times 10^{10}$	✓
(IA)	(8, 4)	$5.1 \times 10^5 < T_R \leq 3.6 \times 10^4$	$7.3 \times 10^6 < M_\phi \leq 9.6 \times 10^6$	✗
(IB)	(4, 2)	$6.0 \times 10^{13} < T_R \leq 5.0 \times 10^7$	$1.0 \times 10^{10} < M_\phi \leq 7.4 \times 10^3$	✗
(IB)	(6, 2)	$6.4 \times 10^{13} < T_R \leq 4.1 \times 10^5$	$8.1 \times 10^7 < M_\phi \leq 0.51 \times 10^1$	✗
(IB)	(8, 4)	$6.4 \times 10^{13} < T_R \leq 3.3 \times 10^3$	$6.6 \times 10^5 < M_\phi \leq 3.4 \times 10^{-5}$	✗
(IC)	(4, 2)	$8.9 \times 10^{12} < T_R \leq 5.0 \times 10^7$	$1.0 \times 10^{10} < M_\phi \leq 5.7 \times 10^4$	✗
(IC)	(6, 2)	$9.0 \times 10^{12} < T_R \leq 4.1 \times 10^6$	$8.1 \times 10^7 < M_\phi \leq 0.36 \times 10^1$	✗
(IC)	(8, 4)	$1.1 \times 10^{12} < T_R \leq 3.3 \times 10^3$	$6.6 \times 10^6 < M_\phi \leq 1.8 \times 10^{-2}$	✗
(IIA)	(4, 2)	$1.3 \times 10^{13} < T_R \leq 5.0 \times 10^8$	$8.0 \times 10^{10} < M_\phi \leq 2.8 \times 10^6$	✗
(IIA)	(6, 2)	$1.2 \times 10^{13} < T_R \leq 4.1 \times 10^6$	$6.5 \times 10^8 < M_\phi \leq 1.8 \times 10^2$	✗
(IIA)	(8, 4)	$1.1 \times 10^{14} < T_R \leq 3.3 \times 10^4$	$5.3 \times 10^6 < M_\phi \leq 1.8 \times 10^{-2}$	✗
(IIB)	(4, 2)	$8.9 \times 10^6 < T_R \leq 5.0 \times 10^8$	$2.0 \times 10^{12} < M_\phi \leq 7.0 \times 10^{13}$	✗
(IIB)	(6, 2)	$8.0 \times 10^6 < T_R \leq 4.1 \times 10^6$	$1.6 \times 10^{11} < M_\phi \leq 9.3 \times 10^9$	✓
(IIB)	(8, 4)	$7.9 \times 10^6 < T_R \leq 3.3 \times 10^4$	$1.3 \times 10^9 < M_\phi \leq 6.3 \times 10^5$	✗
(III)	(4, 2)	$4.0 \times 10^7 < T_R \leq 3.7 \times 10^{10}$	$7.5 \times 10^{11} < M_\phi \leq 7.0 \times 10^{15}$	✗
(III)	(6, 2)	$3.6 \times 10^6 < T_R \leq 4.1 \times 10^9$	$8.2 \times 10^{11} < M_\phi \leq 9.3 \times 10^{14}$	✓
(III)	(8, 4)	$3.5 \times 10^6 < T_R \leq 3.3 \times 10^7$	$6.3 \times 10^9 < M_\phi \leq 6.3 \times 10^{10}$	✓

be consistent in both frames of leptogenesis. This implies that there is a problem with neutrino mass models without  $\theta_{13}$ . Next we study neutrino mass models with nonzero  $\theta_{13}$  and look for consistency in the predictions of two mechanisms of leptogenesis.

## 5. Numerical Analysis and Results with $\theta_{13}$

In this section, we investigate the effects of inclusion of nonzero  $\theta_{13}$  (cf. [1, 2]) in the neutrino mass models and predict the cosmological baryon asymmetry. Unlike in Section 4

TABLE 4

Type	$m_{LL}^{\text{diag}}$	$m_{LL}^0$	$m_{LL} = m_{LL}^0 + \Delta m_{LL}$
QDIA	$\text{diag}(1, -1, 1)m_0$	$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$\begin{pmatrix} x-2y & -ax & -ax \\ -ax & \frac{1}{2}-by & \frac{1}{2}-y \\ -ax & \frac{1}{2}-y & \frac{1}{2}-by \end{pmatrix} m_0$
Input $x = 0.66115$ , $y = 0.16535$ , $m_0 = 0.4$ (for $\tan^2\theta_{12} = 0.45$ , $a = 0.868$ , $b = 1.025$ )			
QDIB	$\text{diag}(1, 1, 1)m_0$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_0$	$\begin{pmatrix} 1-x-2y & ax & ax \\ ax & 1-by & -y \\ ax & -y & 1-by \end{pmatrix} m_0$
Input $x = 8.314 \times 10^{-5}$ , $y = 0.00395$ , $m_0 = 0.4$ eV ( $a = 0.945$ , $b = 0.998$ )			
QDIC	$\text{diag}(1, 1, -1)m_0$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_0$	$\begin{pmatrix} x-2y & -ax & -ax \\ -ax & -by & 1-y \\ -ax & 1-y & -by \end{pmatrix} m_0$
Input $x = 8.211 \times 10^{-5}$ , $y = 0.00395$ , $m_0 = 0.4$ eV ( $a = 0.945$ , $b = 0.998$ )			
IH2A	$\text{diag}(1, 1, 0)m_0$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$\begin{pmatrix} x-2y & -x & -x \\ -x & \frac{1}{2} & \frac{1}{2}-y \\ -x & \frac{1}{2}-y & \frac{1}{2} \end{pmatrix} m_0$
Inverted hierarchy with even CP parity in the first two eigenvalues (IIA), ( $m_i = m_1, m_2, m_3$ ): ( $y/x$ ) = 1.0, $y = 0.005$ , $m_0 = 0.045$ eV			
IH2B	$\text{diag}(1, 1, 1)m_0$	$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} m_0$	$\begin{pmatrix} 1-2y & x & x \\ x & \frac{1}{2} & \frac{1}{2}-y \\ x & \frac{1}{2}-y & \frac{1}{2} \end{pmatrix} m_0$
Inverted hierarchy with odd CP parity in the first two eigenvalues (IIB), ( $m_i = m_1, -m_2, m_3$ ): ( $y/x$ ) = 1.0, $y = 0.6612$ , $m_0 = 0.045$ eV			
NH3	$\text{diag}(0, 0, 1)m_0$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$\begin{pmatrix} 0 & -x & -x \\ -x & 1-x & 1-y \\ -x & 1+x & 1-x \end{pmatrix} m_0$
Inputs are ( $y/x$ ) = 0.0, $x = 0.146$ , $m_0 = 0.028$ eV			

analysis, we do not use the particular form of matrices, but we construct the lightest neutrino mass matrix  $m_{LL}$  using (3) through (5). On substituting the observational values [86] into  $U_{\text{PMNS}}$ , we obtain

$$U_{\text{PMNS}} = \begin{pmatrix} 0.81883 & 0.55230 & 0.156434 \\ -0.48711 & 0.52436 & 0.69840 \\ 0.30370 & -0.64807 & 0.69840 \end{pmatrix}. \quad (25)$$

Using (4), this leads to  $\sin^2\theta_{13} = 0.0244716$ ,  $\tan^2\theta_{12} = 0.45495$ , and  $\tan^2\theta_{23} = 1$ . Then the  $m_{\text{diag}}$  of (5) are obtained from the observation data (cf. [51, 52]) ( $\Delta m_{12}^2 = m_2^2 - m_1^2 = 7.6 \times 10^{-5} \text{ eV}^2$ ,  $\Delta m_{23}^2 = m_2^2 - m_3^2 = 2.4 \times 10^{-3} \text{ eV}^2$ ) and calculated out for normal and inverted hierarchy patterns. The mass eigenvalues  $m_i$  ( $i = 1, 2, 3$ ) can also be taken from [6, 58, 59]. The positive and negative values of  $m_2$  correspond to Type-IA and Type-IB, respectively. Once the matrix  $m_{LL}$  is determined the procedure for subsequent calculations is the same as in Section 4.

Here, we give the result of only the best model due to inclusion of reactor mixing angle  $\theta_{13}$  in predictions of baryon asymmetry, reheating temperature, and inflaton mass ( $M_\phi$ ). Undoubtedly, for  $\tan^2\theta_{12} = 0.45$ , the best model is NH-IA (6, 2) with baryon asymmetry in unflavoured thermal leptogenesis  $B_{uf} = 3.313 \times 10^{-12}$ , single flavoured approximation  $B_{1f} = 8.844 \times 10^{-12}$ , and full flavoured  $B_{3f} = 2.093 \times 10^{-11}$ . If we examine these values, we find that expectedly there is an enhancement in the predictions of baryon asymmetry parameter by a factor of 10 due to inclusion of flavour effects. Similarly in nonthermal leptogenesis, we found that NH-IA is the best model and the predicted results are

$$\begin{aligned} T_R^{\min} &< T_R \leq T_R^{\max} \text{ (GeV)} \\ &= 7.97 \times 10^3 < T_R \leq 4.486 \times 10^6, \\ M_\phi^{\min} &< M_\phi \leq M_\phi^{\max} \text{ (GeV)} \\ &= 8.97 \times 10^8 < M_\phi \leq 2.24 \times 10^{11}. \end{aligned} \quad (26)$$

These results show that the neutrino mass models with  $\theta_{13}$  are consistent in all the three stages of leptogenesis estimations. And normal hierarchy of Type-IA with charged lepton matrix (6, 2) for diagonal form of Drac mass matrix is the most favoured model out of 18 models. And our calculation for all the models either with or without  $\theta_{13}$  shows that it is strong washout  $\tilde{m}_1$  (or  $\tilde{m}$ )  $> m_*$  and  $M_1 \leq 10^{12}$  GeV, the baryon asymmetry is generated at a temperature  $T_R(10^6 \text{ GeV}) < M_1(10^9 \text{ GeV})$  for NH-IA model.

## 6. Conclusions

We have investigated the comparative studies of baryon asymmetry in different neutrino mass models (namely, QDN, IH, and NH) with and without  $\theta_{13}$  for  $\tan^2\theta_{12}$ , and we found that models with  $\theta_{13}$  are better than models without  $\theta_{13}$ . The predictions of any models with zero  $\theta_{13}$  are haphazard in spite of the fact that their predictions are consistent in a piecemeal manner with the observational data (see Tables 2 and 3) whereas the predictions of any models with nonzero  $\theta_{13}$  are consistent throughout the calculations. And among them, only the values of NH-IA (6, 2) satisfied Davidson-Ibarra upper bound on the lightest RH neutrino CP asymmetry  $|\epsilon_1| \leq 3.4 \times 10^{-7}$  and  $M_1$  lies within the famous Ibarra-Davidson bound; that is,  $M_1 > 4 \times 10^8 \text{ GeV}$  [87]. Neutrino mass models either with or without  $\theta_{13}$ , Type-IA for charged lepton matrix (6, 2) in normal hierarchy appears to be the best if  $Y_B^{\text{CMB}} = 6.1 \times 10^{-10}$  is taken as the standard reference value; on the other hand if then charged lepton matrix (5, 2) is not ruled out. We observed that unlike neutrino mass models with zero  $\theta_{13}$ , where  $\mu$  predominates over  $e$  and  $\tau$  contributions, for neutrino mass models with nonzero  $\theta_{13}$ ,  $\tau$  predominates over  $e$  and  $\mu$  contributions. This implies the factor changes for neutrino mass models with and without  $\theta_{13}$ . When flavour dynamics is included the lower bound on the reheated temperature is relaxed by a factor  $\sim 3$  to 10. We also observe enhancement effects in flavoured leptogenesis compared to nonflavoured leptogenesis by one order of magnitude. Such predictions may also help in determining the unknown Dirac Phase  $\delta$  in lepton sector, which we have not studied in the present paper. And our calculations show that the strong washout regime holds which is favoured by the current evidence for neutrino masses; the baryon asymmetry is generated at a temperature  $T_R(10^6 \text{ GeV}) < M_1(10^9 \text{ GeV})$  for NH-IA model. The overall analysis shows that normal hierarchical model appears to be the most favourable choice in nature. Further enhancement from brane world cosmology [88] may marginally modify the present findings, which we have kept for future work.

## Appendix

### Classification of Neutrino Mass Models with Zero

We list here the zeroth order left-handed Majorana neutrino mass matrices  $m_{LL}^0$  [89–92] with texture zeros left-handed

Majorana neutrino mass matrices,  $m_{LL} = m_{LL}^0 + \Delta m_{LL}$ , corresponding to three models of neutrinos, namely, quasi-degenerate (QD1A, QD1B, and QD1C), inverted hierarchical (IH2A, IH2B), and normal hierarchical (NH3) along with the inputs parameters used in each model.  $m_{LL}$  which obey  $\mu$ - $\tau$  symmetry are constructed from their zeroth-order (completely degenerate) mass models  $m_{LL}^0$  by adding a suitable perturbative term  $\Delta m_{LL}$ , having two additional free parameters. All the neutrino mass matrices given in Table 4 predict  $\tan^2\theta_{12} = 0.45$ . The values of three input parameters are fixed by the predictions on neutrino masses and mixings in Table 1.

## Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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## References

- [1] F. P. An, J. Z. Bai, A. B. Balantekin et al., “Observation of electron-antineutrino disappearance at daya bay,” *Physical Review Letters*, vol. 108, Article ID 171803, 2012.
- [2] J. K. Ahn, S. Chebotaryov, J. H. Choi et al., “Observation of reactor electron antineutrinos disappearance in the RENO experiment,” *Physical Review Letters*, vol. 108, no. 19, Article ID 191802, 6 pages, 2012.
- [3] R. Nichol, in *Proceedings of the 25th International Conference on Neutrino Physics and Astrophysics*, Kyoto, Japan, June 2012, <http://neu2012.kek.jp/>.
- [4] D. N. Spergel, L. Verde, H. V. Peiris et al., “First-year Wilkinson microwave anisotropy probe (WMAP)\* observations: determination of cosmological parameters,” *The Astrophysical Journal Supplement Series*, vol. 148, no. 1, p. 175, 2003.
- [5] A. D. Sakharov, “Violation of CP invariance, C asymmetry, and baryon asymmetry of the universe,” *Journal of Experimental and Theoretical Physics Letters*, vol. 5, p. 24, 1967.
- [6] M. Fukugita and T. Yanagida, “Baryogenesis without grand unification,” *Physics Letters B*, vol. 174, no. 1, pp. 45–47, 1986.
- [7] M. A. Luty, “Baryogenesis via leptogenesis,” *Physical Review D*, vol. 45, no. 2, pp. 455–465, 1992.
- [8] M. Flanz, E. A. Paschos, and U. Sarkar, “Baryogenesis from a lepton asymmetric universe,” *Physics Letters, Section B: Nuclear, Elementary Particle and High-Energy Physics*, vol. 345, no. 3, pp. 248–252, 1995.
- [9] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, “On anomalous electroweak baryon-number non-conservation in the early universe,” *Physics Letters B*, vol. 155, no. 1-2, pp. 36–42, 1985.
- [10] E. W. Kolb and M. S. Turner, *The Early Universe*, Addison-Wesley, New York, NY, USA, 1990.



- [11] T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, A. Sawada and A. Sugamoto, Eds., p. 95, KEK, Tsukuba, Japan, 1979.
- [12] M. Gell-Mann, P. Ramond, and R. Slansky, "Complex spinors and unified theories," in *Proceedings of Supergravity Workshop, Stony Brook, New York, 1979*, P. van Nieuwenhuizen and D. Z. Freedman, Eds., pp. 315–321, North-Holland, Amsterdam, The Netherlands, 1979.
- [13] R. N. Mohapatra and G. Senjanovic, "Neutrino mass and spontaneous parity nonconservation," *Physical Review Letters*, vol. 44, p. 912, 1980.
- [14] W. Buchmuller and M. Plumacher, "Neutrino masses and the baryon asymmetry," *International Journal of Modern Physics A*, vol. 15, p. 5047, 2000.
- [15] G. F. Giudice, A. Notari, M. Raidal, A. Riotto, and A. Strumia, "Towards a complete theory of thermal leptogenesis in the SM and MSSM," *Nuclear Physics B*, vol. 685, no. 1–3, pp. 89–149, 2004.
- [16] S. Blanchet and P. Di Bari, "Flavour effects on leptogenesis predictions," *Journal of Cosmology and Astroparticle Physics*, vol. 2007, no. 3, article 18, 2007.
- [17] S. Davidson and A. Ibarra, "A lower bound on the right-handed neutrino mass from leptogenesis," *Physics Letters B*, vol. 535, pp. 25–32, 2002.
- [18] W. Buchmuller, P. Di Bari, and M. Plumacher, "Cosmic microwave background, matter-antimatter asymmetry and neutrino masses," *Nuclear Physics B*, vol. 643, pp. 367–390, 2002.
- [19] T. Hambye and G. Senjanović, "Consequences of triplet seesaw for leptogenesis," *Physics Letters B*, vol. 582, pp. 73–81, 2004.
- [20] E. Ma, N. Sahu, and U. Sarkar, "Low-energy thermal leptogenesis in an extended NMSSM model," *Journal of Physics G*, vol. 34, no. 4, pp. 741–752, 2007.
- [21] W. Buchmuller, R. D. Peccei, and T. Yanagida, "Leptogenesis as the origin of matter," *Annual Review of Nuclear and Particle Science*, vol. 55, pp. 311–355, 2005.
- [22] L. Covi, F. Roulet, and F. Vissani, "CP violating decays in leptogenesis scenarios," *Physics Letters B*, vol. 384, no. 1–4, pp. 169–174, 1996.
- [23] A. Pilaftsis, "Resonant CP violation induced by particle mixing in transition amplitudes," *Nuclear Physics B*, vol. 504, no. 1–2, pp. 61–107, 1997.
- [24] W. Buchmuller and M. Plumacher, "CP asymmetry in Majorana neutrino decays," *Physics Letters B*, vol. 431, pp. 354–362, 1998.
- [25] V. S. Rychkov and A. Strumia, "Thermal production of gravitinos," *Physical Review D*, vol. 75, Article ID 075011, 2007.
- [26] M. Y. Khlopov and A. D. Linde, "Is it easy to save the gravitino?," *Physics Letters B*, vol. 138, no. 4, pp. 265–268, 1984.
- [27] J. Ellis, D. V. Nanopoulos, and S. Sarkar, "The cosmology of decaying gravitinos," *Nuclear Physics, Section B*, vol. 259, no. 1, pp. 175–188, 1985.
- [28] J. Ellis, D. V. Nanopoulos, K. A. Olive, and S.-J. Rey, "On the thermal regeneration rate for light gravitinos in the early universe," *Astroparticle Physics*, vol. 4, no. 4, pp. 371–385, 1996.
- [29] M. Kawasaki and T. Moroi, "Gravitino production in the inflationary Universe and the effects on big-bang nucleosynthesis," *Progress of Theoretical Physics*, vol. 93, no. 5, pp. 879–899, 1995.
- [30] T. Fukuyama, T. Kikuchi, and T. Osaka, "Non-thermal leptogenesis and a prediction of inflaton mass in a supersymmetric SO(10) model," *Journal of Cosmology and Astroparticle Physics*, vol. 6, p. 5, 2005.
- [31] T. Fukuyama and N. Okada, "Neutrino oscillation data versus minimal supersymmetric SO(10) model," *Journal of High Energy Physics*, vol. 2002, no. 11, article 011, 2002.
- [32] M. Kawasaki, K. Kohri, and T. Moroi, "Hadronic decay of late-decaying particles and big-bang nucleosynthesis," *Physics Letters B*, vol. 625, no. 1–2, pp. 7–12, 2004.
- [33] M. Kawasaki, K. Kohri, and T. Moroi, "Big-bang nucleosynthesis and hadronic decay of long-lived massive particles," *Physical Review D*, vol. 71, Article ID 083502, 2005.
- [34] B. Garbrecht, F. Glowacka, and P. Schwaller, "Scattering rates for leptogenesis: damping of lepton flavour coherence and production of singlet neutrinos," *Nuclear Physics B*, vol. 66, p. 89, 2013.
- [35] A. Mazumdar and S. Morisi, "Split neutrinos, two Majorana and one Dirac, and implications for leptogenesis, dark matter, and inflation," *Physical Review D*, vol. 86, no. 4, Article ID 045031, 2012.
- [36] D. Aristizabal Sierra, F. Bazzocchi, I. de Medeiros Varzielas, L. Merlo, and S. Morisi, "Tri/Bi-maximal lepton mixing and leptogenesis," *Nuclear Physics B*, vol. 827, no. 1–2, pp. 34–58, 2010.
- [37] N. Nimai Singh, H. Zeen Devi, and A. Kr Sarma, "Thermal and non-thermal leptogenesis in different neutrino mass models with tribimaximal mixings," <http://arxiv.org/abs/0807.2361>.
- [38] P. F. Harrison and W. G. Scott, "Mu-Tau reflection symmetry in lepton mixing and neutrino oscillations," *Physics Letters B*, vol. 547, no. 3–4, pp. 219–228, 2002.
- [39] C. S. Lam, "Neutrino 2-3 symmetry and inverted hierarchy," *Physical Review D*, vol. 71, no. 9, Article ID 093001, 4 pages, 2005.
- [40] W. Grimus and L. Lavoura, "A three-parameter model for the neutrino mass matrix," *Journal of Physics G: Nuclear and Particle Physics*, vol. 34, no. 7, pp. 1757–1769, 2007.
- [41] A. S. Joshipura and B. P. Kodrani, "Complex CKM matrix, spontaneous CP violation and generalized  $\mu$ - $\tau$  symmetry," *Physics Letters B*, vol. 670, no. 4–5, pp. 369–373, 2009.
- [42] T. Kitabayashi and M. Yasue, "Neutrino oscillations induced by two-loop radiative mechanism," *Physics Letters B*, vol. 490, no. 3–4, pp. 236–241, 2000.
- [43] E. Ma, " $A_4$  symmetry and neutrinos with very different masses," *Physical Review D*, vol. 70, no. 3, Article ID 031901, 5 pages, 2004.
- [44] Y. H. Ahn, S. K. Kang, C. S. Kim, and J. Lee, "Phased breaking of  $\mu$ - $\tau$  symmetry and leptogenesis," *Physical Review D*, vol. 73, Article ID 093005, 2006.
- [45] Y. Koide, "Universal texture of quark and lepton mass matrices with an extended flavor  $2 \leftrightarrow 3$  symmetry," *Physical Review D*, vol. 69, Article ID 093001, 2004.
- [46] Y. Koide, H. Nishiura, K. Matsuda, T. Kukichi, and T. Fukuyama, "Universal texture of quark and lepton mass matrices and a discrete symmetry  $Z_3$ ," *Physical Review D*, vol. 66, Article ID 093006, 2002.
- [47] K. Matsuda and H. Nishiura, "Broken flavor  $2 \leftrightarrow 3$  symmetry and phenomenological approach for universal quark and lepton mass matrices," *Physical Review D*, vol. 73, Article ID 013008, 2006.
- [48] Y. Koide and E. Takasugi, "Neutrino mixing based on mass matrices with a  $2 \leftrightarrow 3$  symmetry," *Physical Review D*, vol. 77, no. 1, Article ID 016006, 7 pages, 2008.
- [49] R. N. Mohapatra, S. Nasri, and H.-B. Yu, "Grand unification of  $\mu$ - $\tau$  symmetry," *Physics Letters, Section B: Nuclear, Elementary*

- Particle and High-Energy Physics*, vol. 636, no. 2, pp. 114–118, 2006.
- [50] K. Nakamura, Particle Data Group et al., “Review of particle physics,” *Journal of Physics G: Nuclear and Particle Physics*, vol. 37, no. 7A, Article ID 075021, 2010.
  - [51] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo, and A. M. Rotunno, “Evidence of  $\theta_{13} > 0$  from global neutrino data analysis,” *Physical Review D*, vol. 84, no. 5, Article ID 053007, 2011.
  - [52] D. V. Forero, M. Tórtola, and J. W. F. Valle, “Global status of neutrino oscillation parameters after Neutrino-2012,” *Physical Review D*, vol. 86, Article ID 073012, 2012.
  - [53] C. Weinheimer and KATRIN Collaboration, “KATRIN, a next generation tritium  $\beta$  decay experiment with sub-eV sensitivity for the electron neutrino mass,” *Progress in Particle and Nuclear Physics*, vol. 48, no. 1, pp. 141–150, 2002.
  - [54] I. Avignone, T. Frank, S. R. Elliott, and J. Engel, “Double beta decay, Majorana neutrinos, and neutrino mass,” *Reviews of Modern Physics*, vol. 80, no. 2, pp. 481–516, 2008.
  - [55] J. Lesgourgues and S. Pastor, “Neutrino mass from cosmology,” *Advances in High Energy Physics*, vol. 2012, Article ID 608515, 34 pages, 2012.
  - [56] P. Ade, N. Aghanim, C. Armitage-Caplan et al., “Planck 2013 results. XVI. Cosmological parameters,” *Astronomy & Astrophysics*, vol. 571, article A16, 66 pages, 2014.
  - [57] G. C. Branco, R. Gonzalez Felipe, and F. R. Joaquim, “Leptonic CP violation,” <http://arxiv.org/abs/1111.5332>.
  - [58] M. A. Luty, “Baryogenesis via leptogenesis,” *Physical Review D*, vol. 45, no. 2, pp. 455–465, 1992.
  - [59] W. Buchmuller, R. D. Peccei, and T. Yanagida, “Leptogenesis as the origin of matter,” *Annual Review of Nuclear and Particle Science*, vol. 55, pp. 311–355, 2005.
  - [60] N. K. Francis and N. Nimai Singh, “Validity of quasi-degenerate neutrino mass models and their predictions on baryogenesis,” *Nuclear Physics B*, vol. 863, no. 1, pp. 19–32, 2012.
  - [61] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, “The effects of topological charge change in heavy ion collisions: ‘event by event P and CP violation,’” *Nuclear Physics A*, vol. 803, no. 3–4, pp. 227–253, 2008.
  - [62] T. Kalaydzhyan and I. Kirsch, “Fluid-gravity model for the Chiral magnetic effect,” *Physical Review Letters*, vol. 106, Article ID 211601, 2011.
  - [63] T. Endoh, S. Kaneko, S. Kang, T. Morozumi, and M. Tanimoto, “Leptogenesis and low energy CP violation, a link,” *Journal of Physics G: Nuclear and Particle Physics*, vol. 29, no. 8, 2003.
  - [64] M. Flanz, E. A. Paschos, and U. Sarkar, “Baryogenesis from a lepton asymmetric universe,” *Physics Letters B*, vol. 345, no. 3, pp. 248–252, 1995.
  - [65] C. H. Albright and S. M. Barr, “Resonant leptogenesis in a predictive SO(10) grand unified model,” *Physical Review D*, vol. 70, Article ID 033013, 2004.
  - [66] J. A. Harvey and M. S. Turner, “Cosmological baryon and lepton number in the presence of electroweak fermion-number violation,” *Physical Review D*, vol. 42, p. 3344, 1990.
  - [67] P. D. Bari, “See-saw geometry and leptogenesis,” *Nuclear Physics B*, vol. 727, no. 1–2, pp. 318–354, 2005.
  - [68] W. Buchmuller, P. D. Bari, and M. Plumacher, “The neutrino mass window for baryogenesis,” *Nuclear Physics B*, vol. 665, pp. 445–468, 2003.
  - [69] K. S. Babu, A. Bachri, and H. Aissaoui, “Leptogenesis in minimal left-right symmetric models,” *Nuclear Physics B*, vol. 738, no. 1–2, pp. 76–92, 2006.
  - [70] G. C. Branco, R. G. Felipe, F. R. Joaquim, and M. N. Rebelo, “Leptogenesis, CP violation and neutrino data: what can we learn?” *Nuclear Physics B*, vol. 640, no. 1–2, pp. 202–232, 2002.
  - [71] E. K. Akhmedov, M. Frigerio, and A. Y. Smirnov, “Probing the seesaw mechanism with neutrino data and leptogenesis,” *Journal of High Energy Physics*, vol. 2003, no. 9, article 021, 2003.
  - [72] B. Adhikary and A. Ghosal, “Nonzero  $U_{e3}$ , CP violation, and leptogenesis in a seesaw type softly broken  $A_4$  symmetric model,” *Physical Review D*, Article ID 073007, 2008.
  - [73] F. Buccella, D. Falcone, and L. Oliver, “Leptogenesis within a generalized quark-lepton symmetry,” *Physical Review D*, vol. 77, Article ID 033002, 2008.
  - [74] A. Abada, H. Aissaoui, and M. Losada, “A model for leptogenesis at the TeV scale,” *Nuclear Physics B*, vol. 728, no. 1–3, pp. 55–66, 2005.
  - [75] O. Vives, “Flavor dependence of CP asymmetries and thermal leptogenesis with strong right-handed neutrino mass hierarchy,” *Physical Review D*, vol. 73, Article ID 073006, 2006.
  - [76] A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada, and A. Riotto, “Flavour issues in leptogenesis,” *Journal of Cosmology and Astroparticle Physics*, vol. 2006, no. 4, article 004, 2006.
  - [77] E. Nardi, Y. Nir, E. Roulet, and J. Racker, “The importance of flavor in leptogenesis,” *Journal of High Energy Physics*, vol. 164, no. 1, pp. 4123–4149, 2006.
  - [78] G. Lazarides and Q. Shafi, “Origin of matter in the inflationary cosmology,” *Physics Letters B*, vol. 258, no. 3, pp. 305–309, 1991.
  - [79] G. F. Giudice, M. Peloso, and A. Riotto, *Journal of High Energy Physics*, vol. 9908, p. 014, 1999.
  - [80] T. Asaka, K. Hamaguchi, M. Kawasaki, and T. Yanagida, “Leptogenesis in inflaton decay,” *Physics Letters B*, vol. 464, no. 1–2, pp. 12–18, 1999.
  - [81] T. Asaka, K. Hamaguchi, M. Kawasaki, and T. Yanagida, “Leptogenesis in an inflationary universe,” *Physical Review D*, vol. 61, Article ID 083512, 2000.
  - [82] T. Asaka, H. B. Nielsen, and Y. Takahashi, “Non-thermal leptogenesis from the heavier majorana neutrinos,” *Nuclear Physics B*, vol. 647, pp. 252–274, 2000.
  - [83] A. Mazumdar, “CMB constraints on non-thermal leptogenesis,” *Physics Letters B*, vol. 580, no. 1–2, pp. 7–16, 2004.
  - [84] T. Fukuyama and N. Okada, “Non-thermal Leptogenesis in a simple 5D SO(10) GUT,” *Journal of Cosmology and Astroparticle Physics*, vol. 9, p. 24, 2010.
  - [85] W. Buchmuller, R. D. Peccei, and T. Yanagida, “Leptogenesis as the origin of matter,” *Annual Review of Nuclear and Particle Science*, vol. 5, pp. 311–355, 2005.
  - [86] S. F. King, “Tri-bimaximal-Cabibbo mixing,” *Physics Letters B*, vol. 718, no. 1, pp. 136–142, 2012.
  - [87] S. Davidson and A. Ibarra, “A lower bound on the right-handed neutrino mass from leptogenesis,” *Physics Letters B*, vol. 535, no. 1–4, pp. 25–32, 2002.
  - [88] N. Okada and O. Seto, “Thermal leptogenesis in brane world cosmology,” *Physical Review D*, vol. 73, Article ID 063505, 2006.
  - [89] G. Altarelli and F. Feruglio, “Neutrino masses and mixings: a theoretical perspective,” *Physics Reports*, vol. 320, pp. 295–318, 1999.

- [90] G. Altarelli and F. Feruglio, “Theoretical models of neutrino masses and mixings,” *Springer Tracts in Modern Physics*, vol. 190, pp. 169–207, 2003.
- [91] S. F. King, “Neutrino mass models,” *Nuclear Physics B—Proceedings Supplements*, vol. 118, pp. 267–276, 2003.
- [92] R. N. Mohapatra, S. Antusch, K. S. Babu et al., “Theory of neutrinos: a white paper,” *Reports on Progress in Physics*, vol. 70, no. 11, p. 1757.



